3C5 Full Technical Report
Attitude Control of spacecraft using Reaction Wheels

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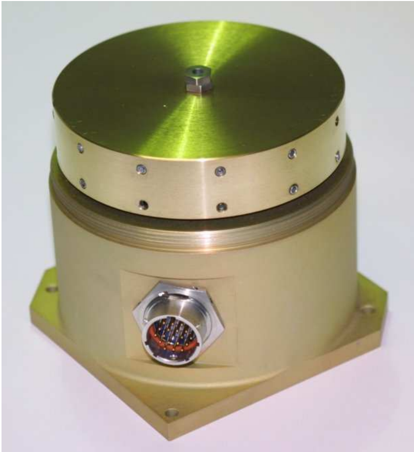
Abstract

Reaction wheels are used to store unwanted angular momentum in satellites and other spacecraft. The equations of motion relating the external and internal torques to the angular velocities and accelerations of the craft and reaction wheels were found. These equations are used to calculate the required torques for fast attitude control of spacecraft.

A comparison with other attitude control systems was made, and practical issues affecting the construction and use of reaction wheels were discussed.

Introduction

Reaction wheels are momentum storage devices used on spacecraft to provide fast attitude control without requiring the use of thrusters. A typical reaction wheel is shown below.



A Typical Reaction Wheel

This report will describe how they work, how they are used, and the practical issues that need to be solved in order to use them effectively.

Background

Many spacecraft have instruments or antennas that must be pointed in a certain direction. Attitude control in necessary to achieve this and to ensure that the craft's orientation is stable. This is particularly important for satellites which use optical equipment. This includes space telescopes, reconnaissance satellites, space exploration satellites and more.

Several methods can be used to provide attitude control for a spacecraft. These are described below.

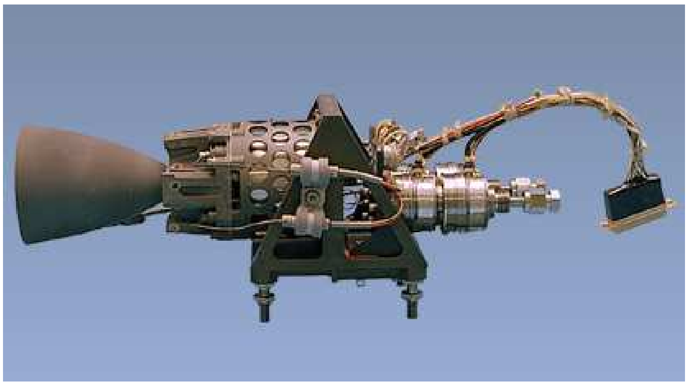
Spin Stabilisation

If a satellite is spun about its major axis (the axis through the centre of mass with the greatest moment of inertia) then its attitude will remain fixed in the absence of external torques. This method was used to stabilise the attitude of the first communication satellite placed in geosynchronous orbit. Of course this only provides stabilisation and not control, so is of no use to many satellites.

One problem with this method of stabilisation that has caused the loss of several satellites is the stability of the spin. In the presence of energy dissipation, only spin about the major axis of a body is stable. Many satellites however are rod shaped, with the axis of symmetry being the minor axis. The solution is to either make the satellite short and fat (more disc-like), so that its axis of symmetry is now also its major axis, or to use dual spin. This involves a spinning rotor connected to a de-spun platform upon which the satellite equipment can be mounted. If the de-spun platform is damped using torsion pendulums then the spin is stable about the satellite's minor axis.

Thrusters

The simplest form of attitude control is to correct the craft's orientation by firing thrusters. At least six are required (two for each axis) though more may be used in practise. It is difficult to achieve accurate attitude control using thrusters as they are highly non-proportional. They are also a drain on the craft's finite supply of fuel.



A Typical Thruster

Momentum Wheels

A two-gimbal momentum wheel can be used instead of thrusters to provide attitude control and stabilisation. The wheel is rotated at high speed and acts like a gyroscope. Disturbance torques can be cancelled by applying torques to the gimbals for two axes, and torquing the wheel itself for the third.

A non-spinning spacecraft with a fixed momentum wheel is effectively a duel-spin spacecraft.

Reaction Wheels

A similar method to using momentum wheels is to use three fixed reaction wheels. These are similar to momentum wheels but have a speed close to zero and are not gimballed. The wheels are usually aligned with the principal axes and each is torqued to cancel disturbance torques on their respective axes. Because their nominal speed is zero they are sometimes called *null momentum wheels*. Reaction wheels can usually spin up to several thousand RPM in either direction.

Reaction and momentum wheels provide much more accurate attitude control than using thrusters alone. This is of particular importance on telescopes and other satellites with cameras (e.g. spy satellites).

The rest of this report will look at the dynamics of reaction wheel controlled spacecraft.

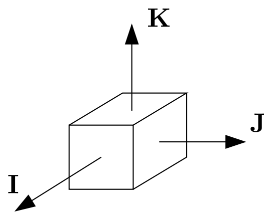
Analysis

While only three reaction wheels are strictly necessary for full control of a spacecraft's attitude, an additional forth wheel is usually used. This wheel is not parallel to any of the others to provide redundancy in case one fails. It also allows the wheels speeds to be changed without affecting their total angular momentum.

Here we will consider a spacecraft with just three wheels a, b, and c, initially at arbitrary orientations.

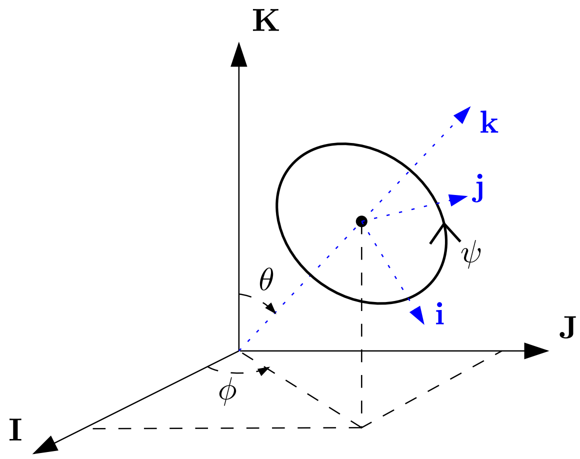
Equations of Motion

A body-fixed coordinate system $\mathbf{I}$, $\mathbf{J}$, $\mathbf{K}$ is oriented with the principal moments of inertia of the spacecraft:



Spacecraft-Fixed Axes

Three reaction wheels a, b, and c are oriented at Euler angles relative to the craft: θ_a, ϕ_a, ψ_a etc. where the subscript indicates the wheel. This will be dropped for some of the discussion. As the wheels can only rotate, *theta* and *phi* are constant.



Reaction Wheel Orientations

Each wheel is an axisymmetric AAC body and has its own coordinate system, $\mathbf{i_a}$, $\mathbf{j_a}$, $\mathbf{k_a}$ etc. These are related to the craft coordinate systems according to an orthogonal rotation matrix for each wheel $[R_a]$, $[R_b]$, $[R_c]$ so that:

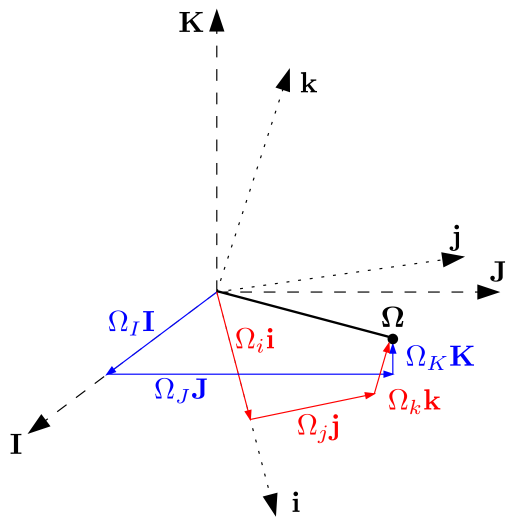
$$\begin{bmatrix} \mathbf{i_a} \\ \mathbf{j_a} \\ \mathbf{k_a} \end{bmatrix} = [R_a] \begin{bmatrix} \mathbf{I} \\ \mathbf{J} \\ \mathbf{K} \end{bmatrix} \quad \text{where} \quad [R_a] = \begin{bmatrix} \cos\theta_a \cos\phi_a & \cos\theta_a \sin\phi_a & -\sin\theta_a \\ -\sin\phi_a & \cos\phi_a & 0 \\ \sin\theta_a \cos\phi_a & \sin\theta_a \sin\phi_a & \cos\theta_a \end{bmatrix}$$

And similarly for wheels b and c.

The angular velocity of the craft is $\mathbf{\Omega}$. This can be converted to the wheel reference frame angular velocities using the $[R]$ rotation matrices.

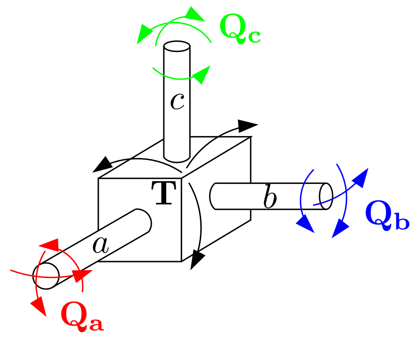
$$\begin{bmatrix} \Omega_{ia} \\ \Omega_{ja} \\ \Omega_{ka} \end{bmatrix} = [R_a] \begin{bmatrix} \Omega_I \\ \Omega_J \\ \Omega_K \end{bmatrix} \quad \text{etc.}$$

Such that $\mathbf{\Omega} = \Omega_I \mathbf{I} + \Omega_J \mathbf{J} + \Omega_K \mathbf{K} = \Omega_i \mathbf{i} + \Omega_j \mathbf{j} + \Omega_k \mathbf{k}$ as shown in the following diagram.



$\boldsymbol{\Omega}$ in two coordinate systems

The resultant torque on the spacecraft is the sum of the external disturbance torque, T and the torque provided by each reaction wheel, Q. Each torque has three components as shown below.



Torques acting on the spacecraft

This affects its angular acceleration according to the following equation:

$$\begin{bmatrix} T_I \\ T_J \\ T_K \end{bmatrix} + \begin{bmatrix} Q_{Ia} \\ Q_{Ja} \\ Q_{Ka} \end{bmatrix} + \begin{bmatrix} Q_{Ib} \\ Q_{Jb} \\ Q_{Kb} \end{bmatrix} + \begin{bmatrix} Q_{Ic} \\ Q_{Jc} \\ Q_{Kc} \end{bmatrix} = \begin{bmatrix} I_{XX} & I_{XY} & I_{XZ} \\ I_{YX} & I_{YY} & I_{YZ} \\ I_{ZX} & I_{ZY} & I_{ZZ} \end{bmatrix} \begin{bmatrix} \dot{\Omega}_I \\ \dot{\Omega}_J \\ \dot{\Omega}_K \end{bmatrix}$$

Now, using the gyroscope equations:

$$\begin{aligned} A_a\dot{\Omega}_{ia} - (A_a\Omega_{ka} - C_a\omega_{ka})\Omega_{ja} &= Q_{ia} \\ A_a\dot{\Omega}_{ja} + (A_a\Omega_{ka} - C_a\omega_{ka})\Omega_{ia} &= Q_{ja} \\ C_a\dot{\omega}_{ka} &= Q_{ka} \end{aligned}$$

And $\dot{\omega}_{ka} = \dot{\Omega}_{ka} + \ddot{\psi}_a$.

The torque in the wheel reference frame can be converted to the craft reference frame using the rotation matrix $[R]$:

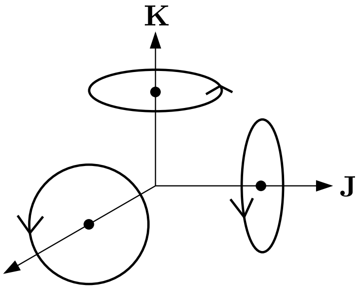
$$\begin{bmatrix} Q_{Ia} \\ Q_{Ja} \\ Q_{Ka} \end{bmatrix} = [R_a] \begin{bmatrix} Q_{ia} \\ Q_{ja} \\ Q_{ka} \end{bmatrix} \quad \text{etc.}$$

Combining all the results so far gives:

$$\begin{bmatrix} T_I \\ T_J \\ T_K \end{bmatrix} + \sum_{a,b,c} [R] \begin{bmatrix} A\dot{\Omega}_i - (A\Omega_k - C\omega_k)\Omega_j \\ A\dot{\Omega}_j + (A\Omega_k - C\omega_k)\Omega_i \\ Q \end{bmatrix} = [I_G] \begin{bmatrix} \dot{\Omega}_I \\ \dot{\Omega}_J \\ \dot{\Omega}_K \end{bmatrix}$$

Q_a, Q_b, and Q_c are the torques applied to the reaction wheels.

To simplify matters we shall assume that the wheels are aligned with the **I J** and **K** axes shown below.



Orthogonal Reaction Wheels

This gives the following $[R]$ matrices:

$$\begin{array}{lll} \theta_a = 0 & \theta_b = \frac{\pi}{2} & \theta_c = \frac{\pi}{2} \\ \phi_a = 0 & \phi_b = 0 & \phi_c = \frac{\pi}{2} \end{array}$$

$$[R_a] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad [R_b] = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad [R_c] = \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Resulting in this overall equation:

$$\begin{bmatrix} T_I \\ T_J \\ T_K \end{bmatrix} + \begin{bmatrix} Q_b \\ Q_c \\ Q_a \end{bmatrix} + \begin{bmatrix} C_a\Omega_J\omega_{aK} + ((A_c - A_a)\Omega_J - C_c\omega_{cJ})\Omega_K + (A_c + A_a)\dot{\Omega}_I \\ -C_a\Omega_I\omega_{aK} + ((A_a - A_b)\Omega_I - C_b\omega_{bI})\Omega_K + (A_b + A_a)\dot{\Omega}_J \\ C_c\Omega_I\omega_{cJ} + ((-A_c - A_b)\Omega_I + C_b\omega_{bI})\Omega_J + (A_c - A_b)\dot{\Omega}_K \end{bmatrix} = [I] \begin{bmatrix} \dot{\Omega}_I \\ \dot{\Omega}_J \\ \dot{\Omega}_K \end{bmatrix}$$

$$\begin{array}{c} \omega_{aK} = \Omega_K + \dot{\psi}_a \\ \dot{\omega}_a = \frac{Q_a}{C_a} \end{array} \quad \text{etc.}$$

This shows that in order to keep $\dot{\mathbf{\Omega}} = \mathbf{\Omega} = \mathbf{0}$, it must be true that $T_I = -Q_b$, $T_J = -Q_c$, and $T_K = -Q_a$. In reality, **T** is unknown and a feedback control system would have to be implemented in order to control the attitude.

Practical Issues

Desaturation

Under the action of a constant disturbance torque (such as from solar wind, or gas leaks), the speed of both momentum and reaction wheels continues to increase until it must be 'desaturated'. This can be done either by firing thrusters or, if the craft is in orbit about the Earth, magnetic torquers may be used.

Zero Speed

As reaction wheels go through their zero speed, they become non-linear due to the increased friction, and loss of lubrication. This may be mitigated either by increasing the torque at this point, or by biasing the wheels so that the always spin in one direction.

Attitude Sensing

In order to control the space craft's attitude, its current attitude must be known.

Pitch and roll can be measured using an Earth sensor. This is accurate to about $0.3 - 1°$. If this is combined with a sun sensor and gyro-compass, three-axis attitude sensing to an accuracy of $0.1 - 0.3°$ can be achieved.

If more accurate attitude sensing is required, a star sensor can be used instead. These can have an accuracy as high as $\pm 0.03°$.

Conclusion

Reaction wheels provide a fuel efficient method for attitude control in satellites and other spacecraft. The equations of motion were derived for a three wheel system at arbitrary and orthogonal orientations, and were found to be non-linear.

Nomenclature

a, b, c	The three reaction wheels.
θ, ϕ, ψ	Orientation of each reaction wheel.
$\dot{\psi}$	Rate of rotation of each reaction wheel relative to the spacecraft.
$[R_{\{a,b,c\}}]$	Rotation matrix between the **IJK** coordinate system and each wheel's **ijk** coordinate system.
$[I_G]$	Inertia matrix of the spacecraft.
$\mathbf{\Omega}$	Spacecraft angular velocity.
$\Omega_{\{I,J,K\}}$	Components of the spacecraft angular velocity in the **IJK** coordinate system.
$\Omega_{\{i,j,k\}\{a,b,c\}}$	Components of the spacecraft angular velocity in each reaction wheel's **ijk** coordinate system.
$\omega_{\{a,b,c\}}$	Angular velocity of each reaction wheel.
$T_{\{I,J,K\}}$	Components of the external torque applied to the spacecraft.
$Q_{\{I,J,K\}}$	Components of each reaction wheel's torque in the **IJK** coordinate system.
$Q_{\{i,j,k\}\{a,b,c\}}$	Components of each reaction wheel's torque in its **ijk** coordinate system.

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